

Efficiency Estimation of Road Transport Safety in Iranian Provinces under Uncertainty Conditions

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Original Article

Abstract

INTRODUCTION: Road safety is a recognized global issue and according to the WHO, road traffic injuries are the eighth leading cause of death in all age groups, especially 5 to 29 years. Therefore, in this article, the road safety performance of Iran's provinces is examined.

METHODS: This research was done using Data Envelopment Analysis (DEA) method which is used in two deterministic and non-deterministic situations in order to evaluate road safety efficiency scores. This method gives scores (inefficiency) that allow road sections to be ranked appropriately in terms of being accident-prone. Uncertainty is one of the inevitable features of real-world problems, for which fuzzy theory and extend the DEA-RS model is used by considering its limitations as probability, necessity, and credibility constraints, and propose three fuzzy models such as possibility of DEA-RS (PosDEA-RS); necessary DEA-RS (NecDEA-RS); and the credibility of the DEA-RS (CreDEA-RS).

FINDINGS: Three models which are extensions of the Data Envelopment Analysis based on the Road Safety (DEA-RS) model are proposed for evaluating road safety performance and the CreDEA-RS model is suitable for assessing the safety of roads in the provinces of Iran.

CONCLUSION: The results show that the provinces located in mountain and forest areas like Gilan have a lower performance in terms of road safety, and provinces located in desert areas like Yazd have a higher road safety performance.

Keywords: Road safety performance; Data Envelopment Analysis (DEA); Possibility theory; Necessity theory; Credibility theory.

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Introduction

Road safety is of great importance for all countries. Road traffic crashes not only impose huge financial losses but, more importantly, cause human life losses. According to the WHO report on road safety, approximately 1.19 million people die annually as a result of road traffic crashes. They highlight that road traffic injuries are the eighth leading cause of death for all age groups and the leading cause of death for people aged 5-29 years. Furthermore, road traffic crashes cost most countries 3% of their gross domestic product. (1) Thus, it is obvious why road safety is an important issue for every country. Policy makers try to improve road safety, and they need to know the

current status of road safety in their region. Consequently, studies have been conducted on evaluating road safety performance.

Tabatabaei et al. (2024) considered accidents according to the environmental, traffic, and geometrical conditions of roads in Iran. A case study was conducted on routes with a length of 144.4 kilometers, resulting in the identification of 154 road sections with different relative risk scores and focuses on the application of Artificial Neural Networks (ANNs) in analyzing road safety. The results reveal the relative importance of different parameters on the weighted index with the ratio of curvature, length of the segment, and condition of the pavement identified as the most influential factors. (2)

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Mansouri et al. (2024) first summarize articles and classify them according to different characteristics (environmental, safety, economic, and energy) and these articles as a basis for developing a novel DEA framework for the evaluation of the efficiency and ranking of road transport systems that also takes into account undesirable outputs, i.e., environmental and safety outputs. As a case study, they evaluate 28 European countries from technical, safety, and environmental aspects. The CCR and SBM models are used to evaluate the efficiency of these countries for the last two years of published data. The results show that Denmark ranks first and Cyprus last for both years. (3)

Andjelković et al. (2024) applied the DEA model to determine the efficiency of 14 road sections according to seven input-output parameters. Seven out of the fourteen alternatives showed full efficiency and were implemented further in the model. After that, the IFRN SWARA method was used for the calculation of the final weights, while IFRN WASPAS was applied for ranking seven of the road sections. The results show that the efficiency values are very stable. According to the results obtained, the best-ranked section is a measuring segment of the Ivanjska–Šargovac section, with a road gradient of -5.5% , which has low deviating values of headways according to the measurement classes from PC-PC to AT-PC, indicating balanced and continuous traffic flow. (4)

Bonera et al. (2023) presented an operational RNS framework for road network screening and safety performance evaluation, and it integrates accident, traffic and road data using a flexible logic, suggesting that road authorities can use this framework to perform a screening strict safety in the road network with the aim of rational planning of safety interventions. (5) Kang et al. (2022) have calculated the road safety performance of the China provinces with output and input criteria using DEA method during the years 2016 to 2018. The results showed that the average road safety performance score of China provinces was 0.657. (6) Fancello et al. (2019) employed Electre III, Concordance Analysis, Vikor and Topsis for identifying the most

critical road sections in a road network and compared the results and claimed that Topsis had the best performance among these methods. (7)

Dabbagh and Ahmadi (2020) have introduced the combined PROMETHEE and ANP method to rank the important indicators of geographic information which safety issues have been ranked first. (8) Chen et al. (2016) applied the Entropy-embedded rank-sum ratio and proposed a methodology for road safety performance benchmarking which made two core activities of the benchmarking into a ‘one-stop’ procedure. (9) Wang & Huang (2016) developed a Bayesian hierarchical joint model for road network safety evaluation and included both micro-level variables (related to road entities and traffic volume) and macro-level variables (socioeconomic, trip generation, and network density variables) (10).

Zamani et al. (2021) has weighted the road safety indicators in the investigation of the situation of Iran's provinces. Due to their results, Qom province was the best and Semnan, Alborz, and Tehran provinces were in the next positions. Meanwhile, the Sistan and Baluchistan province has the most unfavorable relative situation in terms of road safety indicators among the provinces of the country (11). Hong Zhu et al. (2021) have presented a hybrid road safety evaluation model by integrating CEM, regret theory and Weighted Accumulated Product Evaluation to evaluate the road safety performance of Chinese provinces. Then, the entropy method was used to weight the criteria and evaluate the efficiency of road safety in China. The results showed that the average score of road safety efficiency for Eastern, Central and Western regions is gradually decreasing (12).

Nikolaou and Dimitriou (2018) applied DEA and DEA-CEM for analyzing the road safety performance of 23 European Union (13). Shen et al. (2012) used three model extensions of DEA and DEA-RS, the cross-efficiency method, and the categorical DEA model for road safety evaluation. They studied the road safety of 27 European Union countries and identified the reference sets or benchmarks for underperforming countries. (14) Also, Shen et al. (2015) used the DEA-RS model for evaluating road safety and consider the number

of serious injuries in addition to the number of fatalities in their study. (15) Egilmez and McAvoy (2013) proposed a DEA-based Malmquist index model and assessed the productivity of US states in decreasing the number of road fatalities (16). Ganji et al. (2019) developed a novel double-frontier cross-efficiency method for assessing road safety performance and claimed that their proposed double-frontier CEM took into account both optimistic and pessimistic points of view. They used the Evidential Reasoning Approach to reflect the D.M.s' preference structure. (17)

Dabbagh and Nasirifard (2019) have proposed safety as a condition for sustainable development of cities. In critical conditions, the safety of the routes is the main condition of accommodation in critical conditions. (18) Shah et al. (2018) investigate the interaction between road safety risk and influencing factors and used DEA to evaluate road safety risk levels and then applied the Structural Equation Model (SEM) with latent variables to analyze the interaction between the road safety risk level and the latent variables. (19)

Shah et al. (2019) applied the DEA and decision tree (D.T.) to propose a methodology to analyze road safety performance. They used DEA to identify risky and safe segments of a highway and used D.T. to identify the impact of four major factors on the safety level. (20) the road safety performances in provinces of Iran are evaluated using a more realistic version of DEA-based road safety (DEA-RS) model. Uncertainty in inputs and outputs and develop an expanded DEA-RS model is considered, for this, we employed fuzzy theory and considered the constraints of DEA-RS model as possibility, necessity and credibility constraints. Finally, three fuzzy models are proposed. In the following, the three fuzzy models that have been proposed, namely the Possibility DEA-RS model, the Necessity DEA-RS model, and the Credibility DEA-RS model. In Section 3, the inputs and outputs used in the models are described in detail, providing a clear understanding of the variables considered in the evaluation of road safety performance.

The proposed models of this research apply the models to real data obtained from the provinces

of Iran. The results are carefully analyzed and interpreted, shedding light on the road safety performance of different provinces. Finally, section 5 provides a concise summary of the paper's conclusions, highlighting the key findings and implications derived from the study.

Methods

In this research, appropriate input and output indicators will be collected to estimate road safety. The method of collecting information is using statistical yearbooks of the Ministry of Roads and Transport, as well as library studies. The statistical population of this research is the provinces of the country. After collecting the data, the efficiency of Iran's provinces will be calculated using the DEA-RS fuzzy linear programming model. In addition, Lingo software will be used to solve the models.

This section introduces the DEA-based road safety (DEA-RS) model, followed by the introduction of three fuzzy DEA models: Possibility DEA-RS model, Necessity DEA-RS model, and Credibility DEA-RS model.

Preliminaries

In this section, some basic definitions of fuzzy sets are reviewed. (See Yue and Zou (2023) for more details)

Definition 1: The α -cut of the fuzzy set $\tilde{A}$, is the crisp set $\tilde{A}_\alpha = \{x \mid \mu_{\tilde{A}}(x) \geq \alpha\}$

Definition 2: A L-R fuzzy number is expressed as $\tilde{A} = (m, \alpha, \beta)_{LR}$ with the bellow membership function:

$$(1) \quad \mu_{\tilde{A}}(x) = \begin{cases} L(\frac{m-x}{\alpha}) & x \leq m \\ R(\frac{x-m}{\beta}) & x \geq m \end{cases}$$

Where L and R are the left and right functions, respectively, α and β are the non-negative left and right spreads, respectively.

Definition 3: A L-R fuzzy number $\tilde{A} = (m, \alpha, \beta)_{LR} = (m, \alpha, \beta)$ is a triangular fuzzy number if:

$$(2) \quad L(x) = R(x) = \begin{cases} 1-x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Definition 4: Let $\tilde{A} = (m, \alpha, \beta)_{LR}$, $\tilde{B} = (n, \gamma, \delta)_{LR}$ and be two positive triangular fuzzy numbers. Then:

$$(3) \quad \tilde{A} + \tilde{B} = (m, \alpha, \beta)_{LR} + (n, \gamma, \delta)_{LR} = (m+n, \alpha+\gamma, \beta+\delta)_{LR}$$

$$(4) \quad \tilde{A} - \tilde{B} = (m, \alpha, \beta)_{LR} - (n, \gamma, \delta)_{LR} = (m-n, \alpha+\delta, \beta+\gamma)_{LR}$$

Definition 5: A possibility space is defined as $(\Theta, P(\Theta), Pos)$, where Θ is a nonempty set, $P(\Theta)$ is the power set of Θ , and Pos is the possibility measure. Also, X is the universe set. The possibility measure satisfies the below axioms:

- (5) a) $Pos(\emptyset) = 0, Pos(X) = 1$;
 (6) b) $\forall A, B \in P(\Theta), \text{ if } A \subseteq B \Rightarrow Pos(A) \leq Pos(B)$;
 (7) c) $Pos(\bigcup_i A_i) = \max_i \{Pos(A_i)\}$

Definition 6: The necessity measure is defined as $Nec(A) = 1 - Pos(A^c)$. Where A^c is the complementary set of A . The necessity measure satisfies the below axioms:

- (8) a) $Nec(\emptyset) = 0, Nec(X) = 1$;
 (9) b) $\forall A, B \in P(\Theta), \text{ if } A \subseteq B \Rightarrow Nec(A) \leq Nec(B)$;
 (10) c) $Nec(\bigcup_i A_i) = \min_i \{Nec(A_i)\}$

Definition 7: the credibility measure is defined as $Cre(A) = \frac{1}{2} \{Pos(A) + Nec(A)\}$. The credibility measure satisfies the below axioms:

- (11) a) $Cre(\emptyset) = 0, Cre(X) = 1$;
 (12) b) $\forall A, B \in P(\Theta), \text{ if } A \subseteq B \Rightarrow Cre(A) \leq Cre(B)$;
 (13) c) $Cre(A) + Cre(A^c) = 1, \forall A \in P(X)$

Definition 8: Let λ be a fuzzy variable. The possibility, necessity, and credibility of the fuzzy event $(\lambda \geq r)$ are defined as:

- (14) $Pos(\lambda \geq r) = \sup_{t \geq r} \mu_\lambda(t)$
 (15) $Nec(\lambda \geq r) = 1 - Pos(\lambda < r) = 1 - \sup_{t < r} \mu_\lambda(t)$
 (16) $Cre(\lambda \geq r) = \frac{1}{2} \{Pos(\lambda \geq r) + Nec(\lambda \geq r)\}$

DEA-RS model

The input-oriented DEA-VRS model is as follows (21):

$$\begin{aligned} \theta^* = \min \theta \\ \text{s.t.} \\ \sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{i0} \quad i = 1, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} \geq y_{r0} \quad r = 1, \dots, s \\ \sum_{j=1}^n \lambda_j = 1 \\ \lambda_j \geq 0 \quad j = 1, \dots, n \\ \theta \text{ free} \end{aligned} \quad (17)$$

Where n is the number of DMUs, m & s are the numbers of inputs and outputs respectively. x_{ij} is the amount of the i th input for the j th DMU, y_{rj} is the amount of the r th output for the j th DMU. θ denotes the efficiency score of the DMU0. This model is not appropriate for evaluating road

safety since, in the DEA model for evaluating road safety; we want the output- for example, the number of road fatalities- to be as low as possible for the given input levels. In other words, in the DEA-based road safety model, DMUs which have minimum output levels for given input levels are efficient. The DEA-based road safety (DEA-RS) model proposed by Shen et al. (2012) is as follows: (14)

$$\begin{aligned} \theta_0^{DEA-RS} = \min \theta \\ \text{s.t.} \\ \sum_{j=1}^n \lambda_j x_{ij} \geq x_{i0} \quad i = 1, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} \leq \theta y_{r0} \quad r = 1, \dots, s \\ \sum_{j=1}^n \lambda_j = 1 \\ \lambda_j \geq 0 \quad j = 1, \dots, n \\ \theta \text{ free} \end{aligned} \quad (18)$$

Possibility DEA-RS

In this section, the possibility of the DEA model is presented. Then, we present the Possibility of the DEA-RS (PosDEA-RS) model. To develop the DEA-VRS model and present Possibility of DEA, let us prove the following lemma:

Lemma 1: Let $\tilde{x}_1 = (m_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{x}_2 = (m_2, \alpha_2, \beta_2)_{LR}$ be two L-R fuzzy numbers. For a given confidence level $\gamma \in [0, 1]$ it is proven that:

$$(19) \quad Pos(\tilde{x}_1 \geq \tilde{x}_2) \geq \gamma \Rightarrow m_1 + \beta_1 R^{-1}(\gamma) \geq m_2 - \alpha_2 R^{-1}(\gamma)$$

Proof. Suppose that

$$(20) \quad \tilde{x}_1 - \tilde{x}_2 = (m_1, \alpha_1, \beta_1)_{LR} \oplus (-m_2, \beta_2, \alpha_2)_{LR} = (m_1 - m_2, \alpha_1 + \beta_2, \beta_1 + \alpha_2)_{LR} = (\bar{m}, \bar{\alpha}, \bar{\beta})_{LR}$$

Now, we should calculate the crisp equation equivalent to the below equation;

$$(21) \quad Pos(\tilde{x}_1 \geq \tilde{x}_2) = Pos(\tilde{x}_1 - \tilde{x}_2 \geq 0) = Pos(\bar{x} \geq 0) \geq \gamma$$

The below figure shows the fuzzy number $\tilde{x}$:

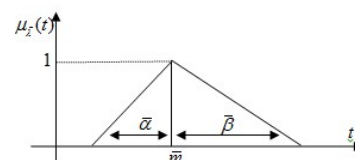


Figure 1. The counterpart PosDEA model

The counterpart PosDEA model can be expressed as follows (Figure 1):

$$\begin{aligned}
\theta_0^{PosDEA} &= \text{Min } \theta \\
s.t.: \\
Pos \left(\sum_{j=1}^n \lambda_j \tilde{x}_{ij} \leq \theta \tilde{x}_{i0} \right) &\geq \gamma_i, \quad i = 1, \dots, m \\
(22) \quad Pos \left(\sum_{j=1}^n \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{r0} \right) &\geq \gamma_r, \quad r = 1, \dots, s \\
\sum_{j=1}^n \lambda_j &= 1 \\
\lambda_j &\geq 0, \quad j = 1, \dots, n \\
\theta &\text{ free}
\end{aligned}$$

The membership functions that we need in model (22) are as follows:

In this study, the data are considered triangular fuzzy numbers. Hence, according to definition 3:

$$(23) \quad L(x) = R(x) = 1 - x \Rightarrow L^{-1}(x) = R^{-1}(x) = 1 - x$$

Necessity DEARS

To develop the DEA-VRS model and present Necessity DEA, let us prove the following lemma.

Lemma 2: Let $\tilde{x}_1 = (m_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{x}_2 = (m_2, \alpha_2, \beta_2)_{LR}$ be two L-R fuzzy numbers. For a given confidence level $\gamma \in [0, 1]$ it is proven that:

$$(24) \quad Nec(\tilde{x}_1 \geq \tilde{x}_2) \geq \gamma \Rightarrow m_1 - \alpha_1 L^{-1}(1 - \gamma) \geq m_2 + \beta_2 L^{-1}(1 - \gamma)$$

According to Figure (1) and Equation (15):

$$(25) \quad Nec(\tilde{x}_1 \geq \tilde{x}_2) = \begin{cases} 1 & 0 < \bar{m} - \bar{\alpha} \\ 1 - L\left(\frac{\bar{m} - \bar{\alpha}}{\bar{\alpha}}\right) & \bar{m} - \bar{\alpha} < 0 < \bar{m} + \bar{\alpha} \\ 0 & \bar{m} - \bar{\alpha} < 0 \end{cases}$$

Then:

$$\begin{aligned}
Nec(\tilde{x}_1 \geq \tilde{x}_2) &= 1 - L\left(\frac{\bar{m} - \bar{\alpha}}{\bar{\alpha}}\right) \geq \gamma \Rightarrow L\left(\frac{\bar{m} - \bar{\alpha}}{\bar{\alpha}}\right) \leq 1 - \gamma \\
(26) \quad \Rightarrow \frac{\bar{m} - \bar{\alpha}}{\bar{\alpha}} &\geq L^{-1}(1 - \gamma) \Rightarrow \frac{m_1 - \alpha_1}{\alpha_1 + \beta_1} \geq L^{-1}(1 - \gamma) \\
&\Rightarrow m_1 - \alpha_1 \geq (\alpha_1 + \beta_1) L^{-1}(1 - \gamma) \\
&\Rightarrow m_1 - \alpha_1 L^{-1}(1 - \gamma) \geq m_2 + \beta_2 L^{-1}(1 - \gamma)
\end{aligned}$$

Credibility DEARS

To develop the DEA-VRS model and present Credibility of DEA, let us prove the following lemma:

Lemma 3: Let $\tilde{x}_1 = (m_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{x}_2 = (m_2, \alpha_2, \beta_2)_{LR}$ be two L-R fuzzy numbers. For a given confidence level $\gamma \in [0, 1]$ it is proven that:

$$(27) \quad \text{If, then: } \gamma \leq \\ Cre(\tilde{x}_1 \geq \tilde{x}_2) \geq \gamma \Rightarrow m_1 + \beta_1 R^{-1}(2\gamma) \geq m_2 - \alpha_2 R^{-1}(2\gamma)$$

$$(28) \quad \text{If } \gamma > 0.5, \text{ then:} \\ Cre(\tilde{x}_1 \geq \tilde{x}_2) \geq \gamma \Rightarrow m_1 - \alpha_1 L^{-1}(2(1 - \gamma)) \geq m_2 + \beta_2 L^{-1}(2(1 - \gamma))$$

Proof. Suppose that $\gamma \leq 0.5$

$$(29) \quad \tilde{x}_1 - \tilde{x}_2 = (m_1, \alpha_1, \beta_1)_{LR} \ominus (m_2, \alpha_2, \beta_2)_{LR} = (m_1 - m_2, \alpha_1 + \beta_2, \alpha_2 + \beta_1)_{LR} = (\bar{m}, \bar{\alpha}, \bar{\beta})$$

Now, we should calculate the crisp equation equivalent to the below equation;

$$Cre(\tilde{x}_1 \geq \tilde{x}_2) = Cre(\tilde{x}_1 - \tilde{x}_2 \geq 0) = Cre(\bar{x} \geq 0) \geq \gamma$$

According to Equation (16):

$$\begin{aligned}
(30) \quad Cre(\bar{x} \geq 0) &= C_r(\bar{x} \geq 0) = \frac{1}{2} [Pos(\bar{x} \geq 0) + Nec(\bar{x} \geq 0)] \\
&= \frac{1}{2} [Pos(\bar{x} \geq 0) + 1 - Pos(\bar{x} < 0)] = \frac{1}{2} [\sup_{t \geq 0} \mu_{\bar{x}}(t) + 1 - \sup_{t < 0} \mu_{\bar{x}}(t)]
\end{aligned}$$

According to Figure 1.

$$(31) \quad Cre(\bar{x} \geq 0) = \begin{cases} 1 & 0 \leq \bar{m} - \bar{\alpha} \\ \frac{1}{2} \left[1 + 1 - L\left(\frac{\bar{m} - \bar{\alpha}}{\bar{\alpha}}\right) \right] & \bar{m} - \bar{\alpha} \leq 0 \leq \bar{m} + \bar{\alpha} \\ \frac{1}{2} \left[R\left(-\frac{\bar{m} - \bar{\alpha}}{\bar{\beta}}\right) + 1 - 1 \right] & \bar{m} \leq 0 \leq \bar{m} + \bar{\alpha} \\ 0 & \bar{m} + \bar{\alpha} < 0 \end{cases} \\
= \begin{cases} 1 & 0 \leq \bar{m} - \bar{\alpha} \\ 1 - \frac{1}{2} L\left(\frac{\bar{m} - \bar{\alpha}}{\bar{\alpha}}\right) & \bar{m} - \bar{\alpha} \leq 0 \leq \bar{m} + \bar{\alpha} \\ \frac{1}{2} R\left(-\frac{\bar{m} - \bar{\alpha}}{\bar{\beta}}\right) & \bar{m} \leq 0 \leq \bar{m} + \bar{\alpha} \\ 0 & \bar{m} + \bar{\alpha} < 0 \end{cases}$$

If $\gamma \leq 0.5$:

$$\begin{aligned}
(32) \quad Cre(\bar{x} \geq 0) \geq \gamma &\Rightarrow \frac{1}{2} R\left(-\frac{\bar{m} - \bar{\alpha}}{\bar{\beta}}\right) \geq \gamma \Rightarrow R\left(-\frac{\bar{m} - \bar{\alpha}}{\bar{\beta}}\right) \geq 2\gamma \\
\frac{\bar{m} - \bar{\alpha}}{\bar{\beta}} &\leq R^{-1}(2\gamma) \Rightarrow -\frac{m_1 - m_2}{\alpha_2 + \beta_1} \leq R^{-1}(2\gamma) \\
&\Rightarrow m_1 - m_2 \leq (\alpha_2 + \beta_1) R^{-1}(2\gamma) \\
&\Rightarrow m_1 + \beta_2 R^{-1}(2\gamma) \geq m_2 - \alpha_2 R^{-1}(2\gamma)
\end{aligned}$$

If $\gamma > 0.5$:

$$\begin{aligned}
(33) \quad Cre(\bar{x} \geq 0) \geq \gamma &\Rightarrow 1 - \frac{1}{2} L\left(\frac{\bar{m} - \bar{\alpha}}{\bar{\alpha}}\right) \geq \gamma \Rightarrow \frac{1}{2} L\left(\frac{\bar{m} - \bar{\alpha}}{\bar{\alpha}}\right) \leq 1 - \gamma \\
L\left(\frac{\bar{m} - \bar{\alpha}}{\bar{\alpha}}\right) &\leq 2(1 - \gamma) \Rightarrow \frac{\bar{m} - \bar{\alpha}}{\bar{\alpha}} \leq L^{-1}(2(1 - \gamma)) \\
&\Rightarrow \frac{m_1 - m_2}{\alpha_1 + \beta_2} \geq L^{-1}(2(1 - \gamma)) \Rightarrow m_1 - m_2 \geq (\alpha_1 + \beta_2) L^{-1}(2(1 - \gamma)) \\
&\Rightarrow m_1 - \alpha_1 L^{-1}(2(1 - \gamma)) \geq m_2 + \beta_2 L^{-1}(2(1 - \gamma))
\end{aligned}$$

The counterpart CreDEA model can be expressed as follows:

$$\begin{aligned}
\theta_0^{CreDEA} &= \text{Min } \theta \\
s.t.: \\
Cre \left(\sum_{j=1}^n \lambda_j \tilde{x}_{ij} \leq \theta \tilde{x}_{i0} \right) &\geq \gamma_i, \quad i = 1, \dots, m \\
(34) \quad Cre \left(\sum_{j=1}^n \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{r0} \right) &\geq \gamma_r, \quad r = 1, \dots, s \\
\sum_{j=1}^n \lambda_j &= 1 \\
\lambda_j &\geq 0, \quad j = 1, \dots, n \\
\theta &\text{ free}
\end{aligned}$$

If $\gamma \leq 0.5$, according to Lemma 3, the first and second constraints of model (35) are expressed as equations (59) and (60), respectively:

$$\begin{aligned}
(36) \quad \theta x_{i0}^m + \theta x_{i0}^{\beta} R^{-1}(2\gamma_i) &\geq \sum \lambda_j x_{ij}^m - \sum \lambda_j x_{ij}^{\alpha} R^{-1}(2\gamma_i) \\
&\Rightarrow \theta x_{i0}^m + \theta x_{i0}^{\beta} (1 - 2\gamma_i) \geq \sum \lambda_j x_{ij}^m - \sum \lambda_j x_{ij}^{\alpha} (1 - 2\gamma_i) \\
&\Rightarrow \sum_{j=1}^n \lambda_j [x_{ij}^m - (1 - 2\gamma_i) x_{ij}^{\alpha}] \leq \theta [x_{i0}^m + (1 - 2\gamma_i) x_{i0}^{\beta}] \\
(37) \quad \sum \lambda_j y_{rj}^{\alpha} + \sum \lambda_j y_{rj}^{\beta} R^{-1}(2\gamma_r) &\geq y_{r0}^{\alpha} - y_{r0}^{\beta} R^{-1}(2\gamma_r) \\
&\Rightarrow \sum \lambda_j y_{rj}^{\alpha} + \sum \lambda_j y_{rj}^{\beta} (1 - 2\gamma_r) \geq y_{r0}^{\alpha} - y_{r0}^{\beta} (1 - 2\gamma_r) \\
&\Rightarrow \sum_{j=1}^n \lambda_j [y_{rj}^{\alpha} + (1 - 2\gamma_r) y_{rj}^{\beta}] \geq y_{r0}^{\alpha} - (1 - 2\gamma_r) y_{r0}^{\beta}
\end{aligned}$$

Thus, if $\gamma \leq 0.5$, the final CreDEA model is as follows:

$$\begin{aligned}
\theta_0^{CreDEA, \gamma \leq 0.5} &= \text{Min } \theta \\
s.t.: \\
\sum_{j=1}^n \lambda_j [x_{ij}^m - (1 - 2\gamma_i) x_{ij}^{\alpha}] &\leq \theta [x_{i0}^m - (1 - 2\gamma_i) x_{i0}^{\beta}] \quad i = 1, \dots, m \\
(38) \quad \sum_{j=1}^n \lambda_j [y_{rj}^{\alpha} + (1 - 2\gamma_r) y_{rj}^{\beta}] &\geq y_{r0}^{\alpha} - (1 - 2\gamma_r) y_{r0}^{\beta} \quad r = 1, \dots, s \\
\sum_{j=1}^n \lambda_j &= 1 \\
\lambda_j &\geq 0, \quad j = 1, \dots, n \\
\theta &\text{ free}
\end{aligned}$$

Thus, if $\gamma \leq 0.5$, the CreDEA-RS model is as follows:

$$\begin{aligned}
 \theta_{0,\gamma \leq 0.5}^{CreDEA-RS} &= \text{Min } \theta \\
 s.t.: \\
 \sum_{j=1}^n \lambda_j [y_{rj}^m - (1-2\gamma_r)y_{rj}^a] &\leq \theta [y_{r0}^m - (1-2\gamma_r)y_{r0}^a] & r=1, \dots, s \\
 \sum_{j=1}^n \lambda_j [x_{ij}^m + (1-2\gamma_i)x_{ij}^a] &\geq x_{i0}^m - (1-2\gamma_i)x_{i0}^a & i=1, \dots, m \\
 \sum_{j=1}^n \lambda_j &= 1 \\
 \lambda_j &\geq 0 & j=1, \dots, n \\
 \theta &\text{ free}
 \end{aligned}
 \quad (39)$$

Thus, if $\gamma > 0.5$, the CreDEA-RS model is as follows:

$$\begin{aligned}
 \theta_{0,\gamma > 0.5}^{CreDEA-RS} &= \text{Min } \theta \\
 s.t.: \\
 \sum_{j=1}^n \lambda_j [y_{rj}^m + (2\gamma_r - 1)y_{rj}^a] &\leq \theta [y_{r0}^m + (2\gamma_r - 1)y_{r0}^a] & r=1, \dots, s \\
 \sum_{j=1}^n \lambda_j [x_{ij}^m - (2\gamma_i - 1)x_{ij}^a] &\geq x_{i0}^m + (2\gamma_i - 1)x_{i0}^a & i=1, \dots, m \\
 \sum_{j=1}^n \lambda_j &= 1 \\
 \lambda_j &\geq 0 & j=1, \dots, n \\
 \theta &\text{ free}
 \end{aligned}
 \quad (40)$$

In this section, the proposed models (namely the Possibility DEA-RS, the Necessity DEA-RS, and the Credibility DEA-RS) are utilized to evaluate road safety in 15 provinces of Iran and are applied to the dataset containing information from the selected provinces. The evaluation process involves analyzing various factors related to road safety performance in each province. The results obtained from the models' application provide insights into the relative performance levels of the provinces in terms of road safety which contribute to a comprehensive understanding of the road safety situation in different regions of Iran.

Input and output factors

In this study, five inputs and three outputs for calculating the efficiency scores of Iran's provinces was applied in terms of road safety. The inputs are passengers per kilometer, tone per kilometer, the length of highways (km), the number of registered automobiles and the number of speed cameras. The outputs are the number of fatalities, the number of injuries, and the number of crashes. The data required for inputs and outputs are obtained from the 2015 annual report released by Iran Road Maintenance & Transportation Organization.

Findings

In this section, the results of evaluating the road safety of Iran provinces are presented. The

proposed fuzzy models without losing any generality are assumed. For sensitivity analysis, the models are implemented for different amounts of γ .

Results of the DEA-RS model

Table 1 shows that 13 provinces have the best performance and acquired an efficiency score equal to 1. In fact, these provinces are known as leading provinces in the field of road safety. Also, Gilan, East Azerbaijan and West Azerbaijan, which have efficiency scores equal to 0.3921, 0.4352, and 0.4405, respectively, have the worst performance. The results can help the policy-makers of this area to improve the poor performance of the province by using better road conditions. For example, the authorities of Gilan province can prevent more accidents and loss of lives and property by building more highways and installing road equipment such as speed cameras.

Results of the PosDEA-RS model

Due to the uncertainty in the real-world data, the previous DEA-RS model using fuzzy inputs and outputs is utilized in this part. It is worth noting that the degree in the model γ is the degree of the possibility of the limitations of the model being established. Without losing the generality of the problem and for simplicity in the previous model, it was assumed that γ the degree is equal for all constraints of the model. The PosDEA-RS model is implemented $\gamma=0.6, 0.8, 1$. Table 2 shows that for $\gamma=1$, the PosDEA-RS is converted to the DEA-RS model, and the results are the same as the results of the DEA-RS model. In fact, $\gamma=1$ the PosDEA-RS model does not consider uncertainty in data. Also, $\gamma=0.6, 0.8$ the provinces of Ilam, Chaharmahal, Bakhtiari, South Khorasan and Hormozgan have the best performance and Gilan has the worst performance. The efficiency scores are reduced as the amount of γ is reduced, so that the average efficiency values decreased from 0.7976 to 0.7509. In addition, none of the provinces have assigned an efficiency score of one which means the increase in resolution power of PosDEA compared to conventional DEA-RS model.

Table 1. The results of the DEA-RS model

Provinces	Efficiency score	Provinces	Efficiency score
East Azerbaijan	0.4332	Fars	1
West Azerbaijan	0.4405	Qazvin	0.7949
Ardebil	0.9261	Qom	0.9558
Isfahan	1	Kurdistan	0.5905
Alborz	0.6744	Kerman	1
Ilam	1	Kermanshah	0.4534
Bushehr	1	Kohgiluyeh & Boyer-Ahmad	0.7966
Tehran	1	Golestan	0.5914

Provinces	Efficiency score	Provinces	Efficiency score
Chaharmahal & Bakhtiari	1	Gilan	0.3921
South Khorasan	1	Lorestan	0.5013
Razavi Khorasan	1	Mazandaran	0.4813
North Khorasan	0.851	Markazi	0.599
Khuzestan	1	Hormozgan	1
Zanjan	0.7024	Hamedan	0.739
Semnan	1	Yazd	1
Sistan & Baluchistan	0.8039	mean	0.7976

Table 2. The results of the PosDEA-RS model

Provinces	Efficiency score			Provinces	Efficiency score		
	$\gamma = 1$	$\gamma = 0.8$	$\gamma = 0.6$		$\gamma = 1$	$\gamma = 0.8$	$\gamma = 0.6$
East Azerbaijan	0.4332	0.4051	0.3794	Fars	1	0.9168	0.8402
West Azerbaijan	0.4405	0.4124	0.3866	Qazvin	0.7949	0.7475	0.7041
Ardebil	0.9261	0.8811	0.8386	Qom	0.9558	0.8848	0.8191
Isfahan	1	0.9375	0.8793	Kurdistan	0.5905	0.5674	0.5451
Alborz	0.6744	0.6447	0.6164	Kerman	1	0.9343	0.8732
Ilam	1	0.9607	0.923	Kermanshah	0.4534	0.4268	0.4019
Bushehr	1	0.9432	0.89	Kohgiluyeh & Boyer-Ahmad	0.7966	0.7624	0.7304
Tehran	1	0.9358	0.8759	Golestan	0.5914	0.5598	0.5308
Chaharmahal & Bakhtiari	1	0.9607	0.923	Gilan	0.3921	0.3682	0.3458
South Khorasan	1	0.9607	0.923	Lorestan	0.5013	0.4733	0.447
Razavi Khorasan	1	0.9239	0.8536	Mazandaran	0.4813	0.4436	0.4118
North Khorasan	0.851	0.8101	0.7714	Markazi	0.599	0.5542	0.5194
Khuzestan	1	0.9009	0.8103	Hormozgan	1	0.9607	0.923
Zanjan	0.7024	0.667	0.6335	Hamedan	0.739	0.6848	0.6377
Semnan	1	0.9436	0.8908	Yazd	1	0.9588	0.9193
Sistan & Baluchistan	0.8039	0.748	0.696	mean	0.7976	0.7509	0.7077

Table 3. The results of the NecDEA-RS model

Provinces	Efficiency score			Provinces	Efficiency score		
	$\gamma = 1$	$\gamma = 0.8$	$\gamma = 0.6$		$\gamma = 1$	$\gamma = 0.8$	$\gamma = 0.6$
East Azerbaijan	0.6248	0.5744	0.5321	Fars	1.2711	1.2107	1.1536
West Azerbaijan	0.6161	0.5755	0.5379	Qazvin	1.1438	1.0579	0.9797
Ardebil	1.1942	1.134	1.0773	Qom	1.4098	1.3036	1.2058
Isfahan	1.2796	1.217	1.037	Kurdistan	0.7262	0.6956	0.6665
Alborz	0.8604	0.8187	0.7793	Kerman	3.2382	3.1102	2.4941
Ilam	1.452	1.3471	1.2497	Kermanshah	0.6178	0.5801	0.545
Bushehr	1.8057	1.6125	1.4369	Kohgiluyeh & Boyer-Ahmad	0.9997	0.9537	0.9101
Tehran	2.0286	1.1739	1.12776	Golestan	0.7828	0.7393	0.6987
Chaharmahal & Bakhtiari	1.3574	1.2755	1.1993	Gilan	0.5412	0.5069	0.475
South Khorasan	2.1222	1.8504	1.6046	Lorestan	0.7785	0.6893	0.6121
Razavi Khorasan	1.06	1.1826	1.1338	Mazandaran	0.7274	0.6687	0.6149
North Khorasan	1.094	1.0395	0.9882	Markazi	0.977	0.8724	0.7931
Khuzestan	1.6211	1.5432	1.4701	Hormozgan	4.4214	3.5786	2.8232
Zanjan	0.9188	0.8674	0.8223	Hamedan	1.0916	1.0091	0.9332
Semnan	2.4269	2.0794	1.7662	Yazd	2.429	2.0816	1.7677
Sistan & Baluchistan	1.1824	1.0903	1.0057	mean	1.3806	1.2399	0.0464

Table 4. The results of the CreDEA-RS model for $\gamma > 0.5$

Provinces	Efficiency Score			Provinces	Efficiency Score		
	$\gamma = 1$	$\gamma = 0.8$	$\gamma = 0.6$		$\gamma = 1$	$\gamma = 0.8$	$\gamma = 0.6$
East Azerbaijan	0.6248	0.5321	0.4639	Fars	1.2711	1.1536	1.0481
West Azerbaijan	0.6161	0.5379	0.4706	Qazvin	1.1438	0.9797	0.848
Ardebil	1.1942	1.0773	0.9736	Qom	1.4098	1.2058	1.0326
Isfahan	1.2796	1.037	1.0497	Kurdistan	0.7262	0.6665	0.6146
Alborz	0.8604	0.7793	0.707	Kerman	3.2382	2.4941	1.444
Ilam	1.452	1.2497	1.0768	Kermanshah	0.6178	0.545	0.4819
Bushehr	1.8057	1.4369	1.132	Kohgiluyeh & Boyer-Ahmad	0.9997	0.9101	0.8325
Tehran	2.0286	1.12776	1.0362	Golestan	0.7828	0.6987	0.625
Chaharmahal & Bakhtiari	1.3574	1.1993	1.0619	Gilan	0.5412	0.475	0.4178
South Khorasan	2.1222	1.6046	1.1815	Lorestan	0.7785	0.6121	0.5326
Razavi Khorasan	1.06	1.1338	1.0631	Mazandaran	0.7274	0.6149	0.522
North Khorasan	1.094	0.9882	0.8941	Markazi	0.977	0.7931	0.6543
Khuzestan	1.6211	1.4701	1.2103	Hormozgan	4.4214	2.8232	1.5412
Zanjan	0.9188	0.8223	0.74	Hamedan	1.0916	0.9332	0.7986
Semnan	2.4269	1.7662	1.2292	Yazd	2.429	1.7677	1.2297
Sistan & Baluchistan	1.1824	1.0057	0.8643	mean	1.3806	0.0464	0.8960

Table 5- The results of the CreDEA-RS model for $\gamma \leq 0.5$

Provinces	Efficiency Score		Provinces	Efficiency Score	
	$\gamma = 0.5$	$\gamma = 0.4$		$\gamma = 0.5$	$\gamma = 0.4$
East Azerbaijan	0.4332	0.4051	Fars	1	0.9168
West Azerbaijan	0.4405	0.4124	Qazvin	0.7949	0.7475
Ardebil	0.9261	0.8811	Qom	0.9558	0.8848
Isfahan	1	0.9375	Kurdistan	0.5905	0.5674
Alborz	0.6744	0.6447	Kerman	1	0.9343
Ilam	1	0.9607	Kermanshah	0.4534	0.4268
Bushehr	1	0.9432	Kohgiluyeh & Boyer-Ahmad	0.7966	0.7624
Tehran	1	0.9358	Golestan	0.5914	0.5598
Chaharmahal & Bakhtiari	1	0.9607	Gilan	0.3921	0.3682
South Khorasan	1	0.9607	Lorestan	0.5013	0.4733
Razavi Khorasan	1	0.9239	Mazandaran	0.4813	0.4436
North Khorasan	0.851	0.8101	Markazi	0.599	0.5542
Khuzestan	1	0.9009	Hormozgan	1	0.9607
Zanjan	0.7024	0.667	Hamedan	0.739	0.6848
Semnan	1	0.9436	Yazd	1	0.9588
Sistan & Baluchistan	0.8039	0.748	mean	0.7976	0.7509

Results of the NecDEA-RS model

The NecDEA-RS model is implemented $\gamma = 0.6, 0.8, 1$. Hormozgan province has the highest efficiency score. (Table 3)

Results of the CreDEA-RS model

The CreDEA-RS model is implemented $\gamma = 0.4, 0.5, 0.6, 0.8, 1$. For $\gamma = 1$, the CreDEA-RS model is converted to the NecDEA-RS model, and for $\gamma = 0.8$, the CreDEA-RS model is converted to the NecDEA-RS model with $\gamma = 0.6$.

The results of the CreDEA-RS model for $\gamma > 0.5$ are shown in Table 4. According to the results, Hormozgan has the best performance, and Kerman and Yazd have the second and third positions, respectively. Gilan, West Azerbaijan and Kermanshah have the weakest efficiency scores. The advantage of this model is its ability to rank DMUs completely in comparison with the DEA-RS model. In other words, In the DEA-RS model, 13 DMUs have obtained the same efficiency score equal to 1, and the DEA-RS model is not able to separate and rank these DMUs in terms of their performance. Like the previous models, the efficiency scores are reduced as the amount of γ is reduced; however, the ranking does not change significantly.

Table 5 shows that for $\gamma = 0.5$, the CreDEA-RS model is converted to the DEA-RS model. In fact, $\gamma = 0.5$, the CreDEA-RS model does not consider uncertainty in data. For $\gamma = 0.4$, the CreDEA-RS model is converted to the PosDEA-RS model $\gamma = 0.8$.

Discussion and Conclusion

This study focuses on evaluating the road safety performance in the provinces of Iran through the utilization of the Data Envelopment Analysis-based Road Safety (DEA-RS) model. The evaluation is conducted under conditions of uncertainty and ambiguity. The article introduces a fuzzy credibility approach to expand the DEA-RS model and proposes a new model with credibility constraints. In the models of this paper, it is assumed that due to the fuzzy structure of the problem data, the constraints can be violated to a certain degree. Then, using the subject of chance constraint planning in fuzzy space, the amount of road safety efficiency of Iran's provinces was calculated. For this purpose, three developed models of possible DEA, mandatory DEA and credit DEA were used for different γ degree values. The underlying concept involves treating the constraints of the DEA-RS model as credibility constraints, leading to the suggestion of a DEA-RS model called CreDEA-RS, specifically designed for assessing road safety in the Iranian provinces. According to the obtained results, different provinces had different performances. The findings indicate that provinces located in mountainous and forested areas, such as Gilan, exhibit significantly lower road safety performance compared to provinces in desert regions like Yazd. Moreover, the results of the proposed model demonstrate that decreasing the value of ' γ ' results in reduced efficiency, without significantly altering the rankings. The results of this research, similar to the findings of Hamedani et al. (2016) indicate that the

northern provinces of the country have a higher ranking in terms of road safety for road transportation, based on factors such as the number of violations, the level of overload, the number of fatalities from accidents, and the distance traveled by passengers. The findings suggest that the Northern provinces have a superior position compared to the southern provinces of Iran, which may be attributed to the findings of Montazer and Nazemfar (2019) in their evaluation of the status and position of Iranian provinces in terms of indicators of road transportation development. The Northern provinces seem to have a better performance in terms of indicators of road transportation development compared to the southern provinces of the country.

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None

Conflict of Interests

The authors declare no conflict of interest.

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